This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1166**

Unique Paper Code : **237601** **G**

Name of the Paper : **Statistical Inference II**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all, selecting

three from each section.

Section I

1. (*a*) Define "Type" I error and power of the test. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against $H_1 : \theta = 1$ on the basis of a single observation from the population with p.d.f. $f(x, \theta) = \theta e^{-\theta x}; x > 0, 0 < \theta < \infty$. Obtain the values of type I and type II errors and the power of the test.

(*b*) What are simple and composite hypotheses ? State and prove Neyman-Pearson lemma for testing a simple hypothesis against a simple alternative. 5,7½

2. (*a*) If W is an MP region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, then prove that it is necessarily unbiased. Also prove that the same holds good if W is an UMP region.

(*b*) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$ where μ is known. Obtain the B.C.R. of size α for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 = \sigma_1^2 (\neq \sigma_0^2)$. Hence obtain the power function of the test. 6½,6

3. (*a*) Given a random sample $X_1, X_2, ... X_n$ from a distribution with p.d.f.

$$f(x, \theta) = \theta e^{-\theta x}, x \geq 0, \theta > 0.$$

Obtain the UMP critical region of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$, in terms of chi-square statistic. Also obtain the power function of the test.

(*b*) Consider n Bernoullian trials with probability of success p for each trial. Derive the likelihood ratio test for testing $H_0 : p = p_0$ against $H_1 : p > p_0$. 6½,6

4. Discuss the method of construction of likelihood ratio test and state its properties. Construct likelihood ratio test for testing the equality of the means of two normal populations with common unknown variance. 12½

Section II

5. Describe S.P.R.T., its OC and ASN functions.

Construct S.P.R.T. for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (0 < \theta_0 < \theta_1)$ on the basis of a random sample drawn from a Poisson distribution with parameter θ. Also obtain its OC and ASN functions. 12½

6. What are the advantages of non-parametric tests ? Define a run and the length of a run. Describe the run test in detail for testing the equality of the two populations. 12½

7. (*a*) Discuss the Mann-Whitney-Wilcoxon test for testing whether two samples are drawn from the same continuous population. How is the test carried out for large samples ? Also discuss the case of ties.

(*b*) Discuss the Kruskal-Wallis test for testing the null hypothesis that K-samples come from same continuous population. Explain the large sample behaviour of the test. 6,6½

8. (*a*) For the S.P.R.T. of strength :

(α_1, β_2) and for given $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$

prove that :

(*i*) $\alpha_1 \leq \frac{\alpha}{1-\beta}, \beta_1 \leq \frac{\beta}{1-\alpha}$

(*ii*) $a_1 + \beta_1 \leq \alpha + \beta$.

(*b*) Describe the sign test, stating clearly the assumptions involved. Also discuss the paired sample sign test. 5,7½